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Pearson Edexcel International Advanced Level

Thursday 9 January 2025

Morning (Time: 1 hour 30 minutes) **Paper reference** **WMA11/01**

Mathematics
International Advanced Subsidiary/Advanced Level
Pure Mathematics P1

You must have: Mathematical Formulae and Statistical Tables (Yellow), calculator	Total Marks
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Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ►

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1. Find

$$\int \left(8x^3 - 6\sqrt{x} - \frac{2}{5x^3} \right) dx$$

Indefinite integration because we have no bounds

giving your answer in simplest form.

(4)

① Change the x values to a more suitable form

$$\int 8x^3 - 6x^{1/2} - 2(5x^{-3}) dx$$

$$\int 8x^3 - 6x^{1/2} - 10x^{-3} dx$$

② Integrate as normal

$$= \frac{8x^{3+1}}{3+1} - \frac{6x^{1/2+1}}{1/2+1} - \frac{10x^{-3+1}}{-3+1} + c$$

Remember to always add +c for indefinite integration

$$= \frac{8x^4}{4} - 4x^{3/2} - \frac{10x^{-2}}{-2} + c$$

$$= 2x^4 - 4x^{3/2} + (5x)^{-2} + c$$

$$\therefore = 2x^4 - 4x^{3/2} + \frac{1}{5x^2} + c$$

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2.

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Given that

- the point A has coordinates $(-2\sqrt{3}, 5)$
- the point B has coordinates $(7\sqrt{3}, 8)$
- the straight line l_1 passes through A and B

- (a) show that the gradient of l_1 is $p\sqrt{3}$, where p is a rational constant to be found.
You must show each step of your working.

(2)

The straight line l_2 is perpendicular to l_1 and passes through A .

- (b) Find the equation of l_2 , giving your answer in the form $y = mx + c$, where m and c are constants.

(3)

a) Find the gradient

$$\text{Point } A = (-2\sqrt{3}, 5)$$

$$\text{Point } B = (7\sqrt{3}, 8)$$

$$\therefore \text{Gradient (m)} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Gradient (m)} = \frac{8 - 5}{7\sqrt{3} - (-2\sqrt{3})} = \frac{\sqrt{3}}{9}$$

$$\text{Gradient (m)} = \frac{\sqrt{3}}{9} \quad \text{where } p = \frac{1}{9}$$

$$\text{Gradient of a Line:}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



Question 2 continued

b) ① Find the gradient of Line 2 - remember it is perpendicular to Line 1

$$L_1 \text{ gradient} = \frac{\sqrt{3}}{9}$$

$$\therefore L_2 \text{ gradient} = \frac{-9}{\sqrt{3}}$$

② Find the equation of Line 2 using Point A

Passes through Point A $(-2\sqrt{3}, 5)$
 x_1 y_1

$$y - y_1 = m(x - x_1)$$

$$y - 5 = \frac{-9}{\sqrt{3}}(x - (-2\sqrt{3}))$$

$$y - 5 = \frac{-9}{\sqrt{3}}(x + 2\sqrt{3})$$

Equation of a Line
 $y - y_1 = m(x - x_1)$

③ You can then simplify and rearrange this further

$$y - 5 = \frac{-9}{\sqrt{3}}x - 18$$

$$y = \frac{-9x}{\sqrt{3}} - 13$$

$$\therefore y = -3\sqrt{3}x - 13$$

$$\therefore \text{Equation of Line 2: } y = -3\sqrt{3}x - 13$$

(Total for Question 2 is 5 marks)



3. The population of a town was monitored.

Exactly 5 years after monitoring began, the population was 58 000 $\Rightarrow 58$

Exactly 10 years after monitoring began, the population was 65 000 $\Rightarrow 65$

Given that the population of the town, P thousand, t years after monitoring began can be modelled by the equation

$$P^2 = a + bt^3$$

where a and b are constants,

- (a) find the value of a and the value of b .

(3)

According to the model, exactly T years after monitoring began, the population was 85 000

Making your method clear,

- (b) find the value of T , giving your answer to one decimal place.

(Solutions relying entirely on calculator technology are not acceptable.)

(2)

- a) ① To find a and b , we must form 2 simultaneous equations

$$P^2 = a + bt^3$$

$$\textcircled{1} \quad - 58^2 = a + b(5)^3$$

$$3364 = a + 125b$$

$$\textcircled{2} \quad - 65^2 = a + b(10)^3$$

$$4225 = a + 1000b$$

- ② Solve the 2 simultaneous equations

$$(3364 = a + 125b) \times 8$$

$$\hookrightarrow 26912 = 8a + 1000b$$

$$\therefore 26912 = 8a + 1000b$$

$$4225 = a + 1000b$$

$$\hline 22687 = 7a + 0$$

$$\therefore a = 3241$$



Question 3 continued

③ Substitute our value for a back into any equation to find b

$$a = 3241$$

$$\textcircled{1} - 3364 = a + 125b$$

$$3364 = 3241 + 125b$$

$$\therefore 123 = 125b$$

$$\therefore b = 0.984$$

$$\therefore a = 3241 \text{ and } b = 0.984$$

b) ① Substitute our values for a, b and the population back into the original equation

$$P^2 = a + bt^3$$

$$85^2 = 3241 + 0.984t^3$$

② Solve for t

$$7225 = 3241 + 0.984t^3$$

$$3984 = 0.984t^3$$

$$t^3 = 4048.780\dots$$

$$t = 15.9$$

$$\therefore T = 15.9 \text{ years}$$

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4. In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

(i) Given that

$$y = a^x \quad \text{where } a \text{ is a positive constant}$$

express, in simplest form, in terms of y and a

(a) a^{3x+1} (1)

(b) $\frac{5}{(3a^{1-x})^{-2}}$ (3)

(ii) (a) Use the substitution $p = 9^t$ to show that the equation

$$3(3^{4t+2} + 1) = 82 \times 9^t$$

can be rewritten as

$$27p^2 - 82p + 3 = 0$$
 (2)

(b) Hence solve

$$3(3^{4t+2} + 1) = 82 \times 9^t$$
 (3)

i) a) ① Use the indices rule to split the power

$$\begin{aligned} a^{3n+1} &= a^{3n} \times a^1 \\ &= a^{3n} \times a \\ &\downarrow \end{aligned}$$

Indices Rule:
 $a^{m+n} = a^m \times a^n$

② Rewrite a^{3n} using the indices rule

$$a^{3n} \rightarrow (a^3)^n$$

Indices Rule:
 $(a^m)^n = a^{mn}$

We know: $y = a^2$

$$\therefore (a^3)^n = y^3$$

And then combine

$$a^{3n} \times a^1$$

↓

$$y^3 \times a \quad \therefore = ay^3$$



Question 4 continued

b) ① Bring the denominator to the top

$$\frac{5}{(3a^{1-x})^{-2}} \rightarrow 5 (3a^{1-x})^2$$

Indices	Rule:
A^{-2}	$= \frac{1}{A^2}$

② Square the brackets

$$= 9a^{2-2x}$$

③ Multiply by 5

$$5 \times (9a^{2-2x})$$

$$= 45a^{2-2x}$$

④ Rewrite everything using $y = a^x$

$$45a^{2-2x}$$

$$a^2 \times a^{-2x}$$

$$a^{-2x} = \frac{1}{(a^x)^2}$$

$$= \frac{1}{y^2} \times 45a^2$$

$$\therefore = \frac{45a^2}{y^2}$$

ii) a) ① Rewrite powers using base 9

$$9 = 3^2$$

$$\therefore q^t = 3^{2t}$$

$$\therefore p = 3^{2t}$$

② Rewrite 3^{4t+2}

$$3^{4t} \times 3^2$$

$$\downarrow \quad \downarrow$$

$$p^2 \quad 9$$

$$\therefore = 9p^2$$

③ Substitute into the equation

$$3(3^{4t+2} + 1) = 82 \times q^t$$

$$\text{Expand } 3(9p^2 + 1) = 82p$$

$$\hookrightarrow 27p^2 + 3 - 82p = 0$$

$$0 = 27p^2 - 82p + 3$$

Question 4 continued

b) ① Now we have to solve the normal quadratic first

$$27p^2 - 82p + 3 = 0$$

$$(p-3)(27p-1) = 0$$

$$\therefore p = 3 \text{ or}$$

$$p = \frac{1}{27}$$

② Substitute into $p = 9^t$ to find t

$$p = 9^t$$

$$3 = 9^t$$

$$\frac{1}{27} = 9^t$$

$$3^1 = (3^2)^t$$

$$\frac{1}{27} = (3^2)^t$$

$$\therefore 1 = 2t$$

$$\therefore t = \frac{1}{2}$$

$$\frac{1}{3^3} = 3^{-3}$$

$$\therefore -3 = 2t$$

$$\therefore t = -\frac{3}{2}$$

$$\therefore t = \frac{1}{2} \text{ or } t = -\frac{3}{2}$$

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5. In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.

The curve C has equation

Differentiation

$$y = 4x^3 + \frac{2}{x} + 9 \quad x > 0$$

(a) Find $\frac{dy}{dx}$, giving your answer in simplest form. (2)

Given that

- the point P lies on C
- the line with equation $y = k - 5x$, where k is a constant, is the tangent to C at P

(b) show that the x coordinate of P satisfies the equation

$$12x^4 + 5x^2 - 2 = 0 \quad (2)$$

(c) Hence find the value of k . (4)

a) ① Rewrite the equation to make it easier to differentiate

$$y = 4x^3 + \frac{2}{x} + 9$$

$$y = 4x^3 + 2x^{-1} + 9$$

② Differentiate

$$\frac{dy}{dx} = 3(4x^2) + (-1)(2x^{-2})$$

$$\therefore \frac{dy}{dx} = 12x^2 - 2x^{-2}$$

General Differentiation:
 $\frac{d}{dx} ax^n = nax^{n-1}$

b) ① Find the gradient

$$y = k - 5x$$

$$\therefore \text{gradient } \left(\frac{dy}{dx}\right) = -5$$

② Equate $\frac{dy}{dx}$ to -5 and simplify

$$\therefore 12x^2 - 2x^{-2} = -5$$

$$12x^2 - \frac{2}{x^2} = -5x$$

$$(x^2) \quad x^2 \quad (x^2)$$



Question 5 continued

$$\therefore 12x^4 - 2 = -5x^3$$

$$12x^4 + 5x^3 - 2 = 0$$

c) ① Find the x values first

Factorise $12x^4 + 5x^3 - 2 = 0$

$$(4x^2 - 1)(3x^2 + 2) = 0$$

$$\therefore x^2 = \frac{1}{4} \quad \text{or} \quad x^2 = \frac{-2}{3} \quad \times \quad (\text{not needed because } x > 0)$$

$$\therefore x = \frac{1}{2}$$

② Find the corresponding y value to the positive x coordinate we found above using the equation of curve C

$$y = 4\left(\frac{1}{2}\right)^3 + \frac{2}{\left(\frac{1}{2}\right)} + 9$$

$$\therefore y = \frac{27}{2}$$

③ To find k, substitute back into the equation $y = k - 5x$

$$y = k - 5x$$

$$\frac{27}{2} = k - 5\left(\frac{1}{2}\right)$$

$$\therefore \frac{27}{2} = k - \frac{5}{2}$$

$$\therefore k = 16$$

$$\therefore k = 16$$

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6. In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation $y = f(x)$, $x > 0$

Given that

- the point $P(4, -5)$ lies on C
- $f'(x) = \frac{2x^2 + ax + b}{4\sqrt{x}}$, where a and b are constants
- the gradient of the tangent to C at P is 7

(a) show that

$$\textcircled{1} \quad 4a + b = 24 \tag{2}$$

Given also that $a + b = -9$ — $\textcircled{2}$

(b) find, in simplest form, $f(x)$ (7)

Curve C is transformed to the curve with equation $y = f(x - 3)$

Given that point P is transformed to the point Q ,

(c) state the coordinates of Q . (1)

a) $\textcircled{1}$ Equate $f'(x)$ which is the gradient to 7

$$\frac{2x^2 + ax + b}{4\sqrt{x}} = 7$$

$\textcircled{2}$ Sub in the x coordinate of $P(4, -5)$ into $f'(x) = 7$

$$\frac{2(4)^2 + a(4) + b}{4\sqrt{4}} = 7$$

$$\therefore 32 + 4a + b = 7 \times 8$$

$$32 + 4a + b = 56$$

$$4a + b = 24$$

$$\therefore 4a + b = 24 \quad \text{Thus proven}$$

b) $\textcircled{1}$ Solve the simultaneous equations to find a and b

$$1 - \quad 4a + b = 24$$

$$2 - \quad a + b = -9$$

$$3a = 33$$

$$\therefore a = 11$$

when $a = 11$

$$11 + b = -9 \quad \therefore b = -20$$



Question 6 continued

② Substitute our values of a and b back into the equation $f'(x)$

$$f'(x) = \frac{2x^2 + 11x - 20}{4\sqrt{x}}$$

③ Rewrite $f'(x)$ and then integrate to find the equation of Curve C

$$f'(x) = \frac{2x^2}{4x^{1/2}} + \frac{11x}{x^{1/2}} - \frac{20}{x^{1/2}}$$

$$f'(x) = \frac{1}{2}x^{3/2} + \frac{11x^{1/2}}{4} - 5x^{-1/2}$$

$$\int \frac{1}{2}x^{3/2} + \frac{11x^{1/2}}{4} - 5x^{-1/2}$$

$$y = \frac{1}{5}x^{5/2} + \frac{11}{6}x^{3/2} - 10x^{1/2} + c$$

Remember to always add +c for indefinite integration

④ To work out c, substitute in Point (4, -5) into our new equation

$$-5 = \frac{1}{5}(4)^{5/2} + \frac{11}{6}(4)^{3/2} - 10(4)^{1/2} + c$$

$$\begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ \frac{32}{5} & \frac{44}{3} & -20 \end{array}$$

$$\therefore -5 = \frac{32}{5} + \frac{44}{3} - 20 + c$$

$$\therefore c = -5 - \frac{16}{15}$$

$$c = \frac{-91}{15}$$

$$\therefore f(x) = \frac{1}{5}x^{5/2} + \frac{11}{6}x^{3/2} - 10x^{1/2} - \frac{91}{15}$$

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Question 6 continued

c) ① Find out what the transformation is

$$y = f(x-3)$$

→ Graph moves to the right by 3 units

$$\text{Point } P(4, -5) \longrightarrow (4+3, -5) = (7, -5)$$

$$\therefore Q = (7, -5)$$

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7. In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$y = \frac{2}{x} - k$$

where k is a **positive** constant.

- (a) Sketch the graph of C .

Show on your sketch

- the coordinates of any points of intersection of C with the coordinate axes
- the equation of the horizontal asymptote to C

stating each in terms of k .

(3)

The line l has equation $y = -kx - 6$

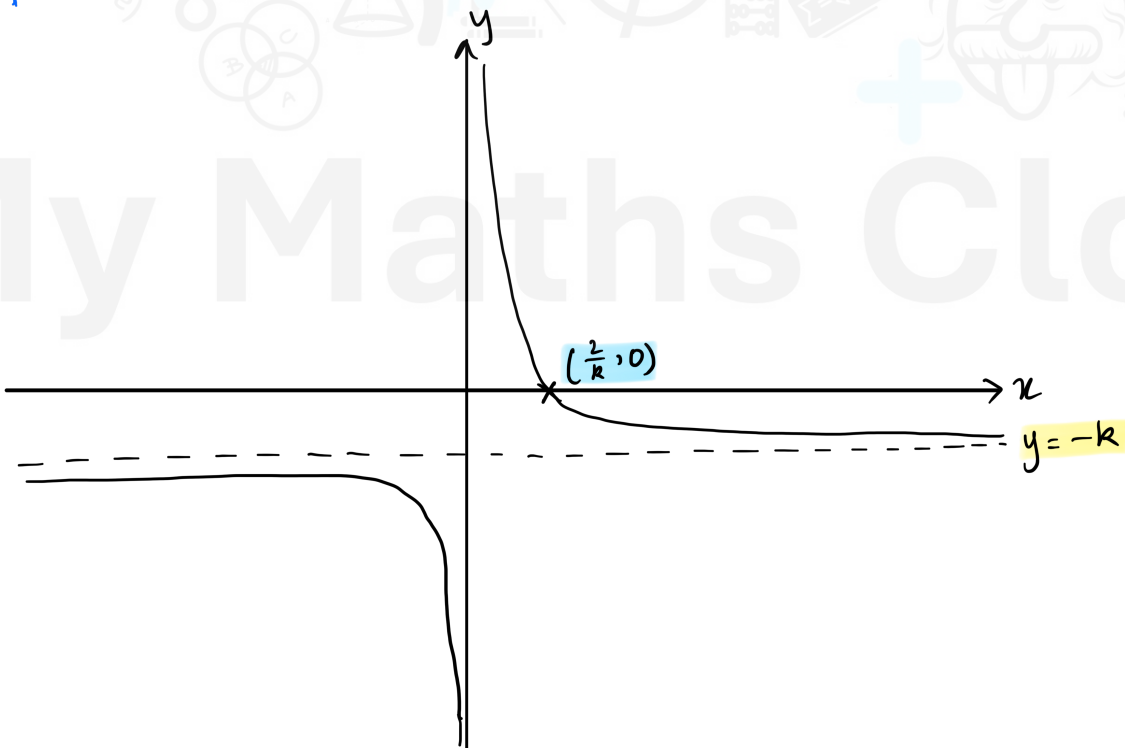
Given that l intersects C at 2 distinct points,

- (b) find the range of possible values of k .

(5)

Part A:

*



Question 7 continued

a) ① Find any point of intersections with the x and y axis

when $y=0$

$$0 = \frac{2}{x} - k$$

$$k = \frac{2}{x}$$

$$\therefore x = \frac{2}{k}$$

when $x=0$

$$y = \frac{2}{0} - k$$

0 This is not possible, therefore there are no intersections with the y axis

but we can say there

is a horizontal asymptote at $y = -k$

② Draw the graph (as shown above) *

b) ① To find any Points of Intersections, we must first equate the 2 equations and simplify

$$y = \frac{2}{x} - k \quad \text{and} \quad y = -kx - b$$

$$(x) \frac{2}{x} - k = -kx - b$$

$$2 - kx = -kx^2 - bx$$

$$\therefore kx^2 + \underbrace{2 - kx + bx}_{x(b-k)} = 0$$

$$\therefore kx^2 + (b-k)x + 2 = 0$$

② As there are 2 points of intersections, we can use the discriminant $b^2 - 4ac > 0$

$$\underbrace{kx^2}_a + \underbrace{(b-k)x}_b + \underbrace{2}_c = 0$$

$$b^2 - 4ac > 0$$

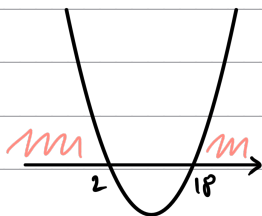
$$(b-k)^2 - 4(k)(2) > 0$$

$$k^2 - 12k + 36 - 8k > 0$$

$$k^2 - 20k + 36 > 0$$

$$(k-18)(k-2) > 0$$

$$\therefore k > 18 \text{ or } k < 2$$



$$\therefore k > 18 \text{ or } k < 2$$

2 Intersection Points:
 $b^2 - 4ac > 0$

8. Remember to keep your calculator in radians for this question!

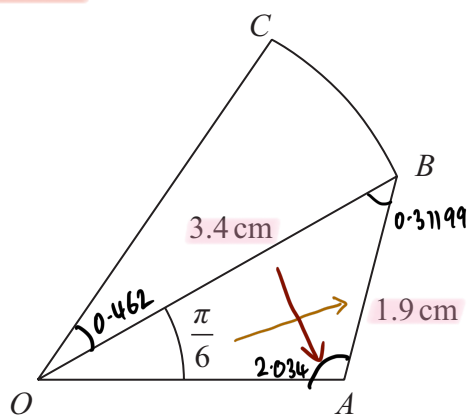


Figure 1

Figure 1 shows a sketch of a design for a badge. The design consists of a triangle OAB joined to a sector OBC of a circle with centre O . In the design

- $OB = 3.4$ cm
- $AB = 1.9$ cm
- angle $AOB = \frac{\pi}{6}$ radians
- angle $OAB > \frac{\pi}{2}$ radians

Making your method clear,

- find the size of angle OAB , giving your answer in radians to 4 significant figures, (3)
- find the area of triangle OAB , in cm^2 , giving your answer to 3 significant figures. (2)

Given that the ratio of the area of sector OBC to the area of triangle OAB is 3 : 2

- show that angle BOC is 0.462 radians to 3 significant figures. (3)
- Hence find the perimeter of the badge, in cm, to the nearest integer. (5)

a) ① As you have an angle-side-side, you can use the sine rule to work out angle OAB

$$\frac{1.9}{\sin \frac{\pi}{6}} = \frac{3.4}{\sin OAB}$$

Sine Rule:

$$\frac{A}{\sin a} = \frac{B}{\sin b}$$

$$\therefore \sin OAB = \frac{\sin \frac{\pi}{6} \times 3.4}{1.9}$$

$$\sin OAB = 0.894\dots$$

$$\therefore OAB = 1.1078 \text{ radians}$$

$$\therefore OAB = 1.108 \text{ rad}$$



Question 8 continued

② But, remember the sine ambiguity! $\sin \theta = \sin(\pi - \theta)$

So another angle with the same sine is $\pi - 1.108$

③ Now calculate the second angle

$$\pi = 3.142$$

$$\therefore 3.142 - 1.108 = 2.034$$

$$\therefore \text{Angle OAB} = \underline{\underline{2.034}}$$

b) ① Lets first work out the angle OBA

$$(180^\circ) \pi = \frac{\pi}{6} + 2.034 + \text{OBA}$$

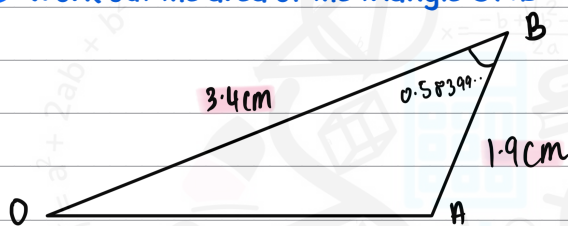
$$\therefore \text{OBA} = \pi - \frac{\pi}{6} - 2.034$$

$$\text{OBA} = 0.58399\dots$$

Radians:

$$\pi = 180^\circ$$

② Work out the area of the triangle OAB



Area of Triangle:

$$\text{Area} = \frac{1}{2} \times a \times b \times \sin C$$

$$\therefore \text{Area} = \frac{1}{2} \times 3.4 \times 1.9 \times \sin 0.58399$$

$$= 1.78$$

$$\therefore \text{Area} = \underline{\underline{1.78 \text{ cm}^2}}$$

c) ① First, we have to work out the area of sector OBC using the ratio 3:2

Area of Sector OBC : Area of Triangle OAB

$$\begin{matrix} \times 0.89 & \left\{ \begin{matrix} 3 \\ = 2.67 \text{ cm}^2 \end{matrix} \right. & : & \begin{matrix} 2 \\ 1.78 \end{matrix} & \left. \right\} \times 0.89 \end{matrix}$$

② We can now work out the angle BOC using the area of sector formula in radians

$$\therefore 2.67 = \frac{1}{2} \times 3.4^2 \times \theta$$

$$\therefore 2.67 = 5.78 \times \theta$$

$$\therefore \theta = 0.46193\dots \text{ radians}$$

Area of Sector:

$$\text{Area} = \frac{1}{2} r^2 \theta$$

$\therefore \text{Angle BOC} = \underline{\underline{0.462 \text{ radians}}}$ Thus proven.

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Question 8 continued

d) ① First, let's write out all the lengths needed for our perimeter and identify which lengths we have

$$\text{Perimeter} = OC + CB + BA + AO$$

↓
x
↓
x

3.4
1.9

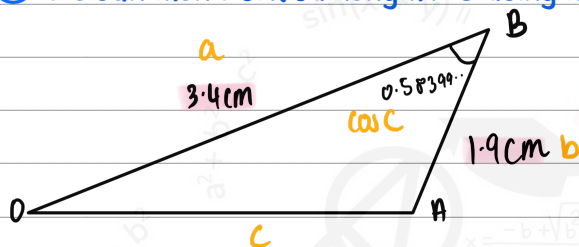
② Let's work out length CB

$$CB = 3.4 \times 0.462$$

$$= 1.5708 \text{ cm}$$

Arc length in Radians
 $L = r\theta$

③ We can then work out length AO using the cosine rule



Cosine Rule:
 $c^2 = a^2 + b^2 - 2ab \cos C$

$$\therefore AO^2 = 3.4^2 + 1.9^2 - 2(3.4)(1.9) \cos 0.588399\dots$$

$$AO^2 = 4.422\dots$$

$$\therefore AO = 2.1 \text{ cm}$$

④ We can now work out the total perimeter

$$\text{Perimeter} = OC + CB + BA + AO$$

$$= 3.4 + 1.57 + 1.9 + 2.1$$

$$= 8.97 \text{ cm}$$

$$\therefore \text{Perimeter} = \underline{\underline{9 \text{ cm}}}$$



9.

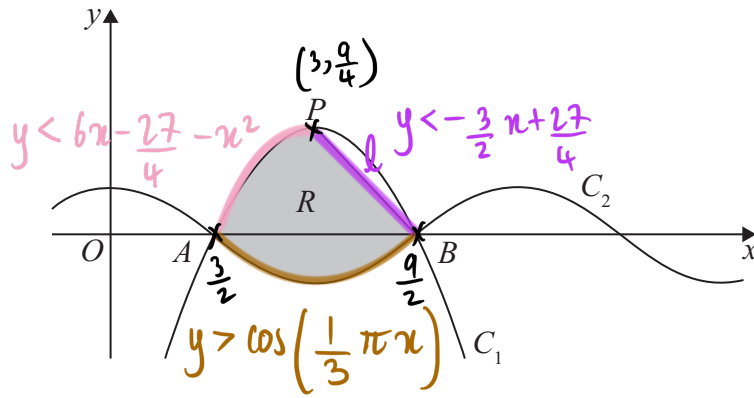


Figure 2

- (a) Express $6x - \frac{27}{4} - x^2$ in the form $a + b(x + c)^2$ where a , b and c are constants to be found. **Completing the square** (3)

Figure 2 shows part of a sketch of curve C_1 with equation

$$y = 6x - \frac{27}{4} - x^2$$

Given that the point P is the maximum point on C_1

- (b) state the coordinates of P (2)

Figure 2 also shows part of a sketch of curve C_2 with equation

$$y = \cos(kx)$$

where k is a constant and x is measured in radians.

Given that C_1 and C_2 intersect on the x -axis at point A and at point B , as shown in Figure 2,

- (c) (i) state the x coordinate of B
 (ii) state the value of k
 (iii) state the period of C_2 (3)
Horizontal distance before the graph repeats itself

The line segment L joins P and B .

The region R , shown shaded in Figure 2, is bounded by L , C_1 and C_2

- (d) Use inequalities to define R . (5)

a) **We have to complete the square**

$$6x - \frac{27}{4} - x^2$$

$$-x^2 + 6x - \frac{27}{4}$$



Question 9 continued

Take out the -

$$-(x^2 - 6x) - \frac{27}{4}$$

$$-[(x-3)^2 - 9] - \frac{27}{4}$$

Multiply the - back in:

$$-(x-3)^2 + 9 - \frac{27}{4}$$

$$-(x-3)^2 + \frac{9}{4}$$

$$\therefore \frac{9}{4} - (x-3)^2 \quad \text{where } a = -3$$

$$b = -1$$

$$c = \frac{9}{4}$$

b) The maximum point is also called the turning point which we can work out from the equation:

$$\frac{9}{4} - (x-3)^2$$

$$\therefore \text{Maximum point} = \left(3, \frac{9}{4}\right)$$

c) i) ① We can work out the x coordinate of Curve C₁ when y = 0

$$y = 6x - \frac{27}{4} - x^2$$

$$0 = -x^2 + 6x - \frac{27}{4}$$

$$\therefore (2x-9)(2x-3) = 0$$

$$\therefore x = \frac{9}{2} \quad \text{or} \quad x = \frac{3}{2} \quad (\text{from diagram})$$

B

A

$$\therefore B = \frac{9}{2}$$

ii) ① Identify the change from the normal cosine graph to the y = cos(kx) graph

$$\text{Curve } C_2 \Rightarrow y = \cos(kx)$$

② Point B is when the graph crosses the x axis, and therefore cosine is 0 when:

$$kx = \frac{\pi}{2}, \frac{3\pi}{2}$$

sub in B $\left(\frac{9}{2}\right)$

$$k\left(\frac{9}{2}\right) = \frac{3\pi}{2}$$

$$\therefore k = \frac{1}{3}\pi$$

$$\therefore k = \frac{\pi}{3}$$



Question 9 continued

iii) ① The period of curve C is $\frac{2\pi}{k}$

② Substitute in our value of k

$$\frac{2\pi}{\frac{1}{3}\pi} = 6$$

$$\therefore \text{Period} = 6$$

d) ① Work out the equation line segment L

L joins $P(3, \frac{9}{4})$ and $B(\frac{9}{2}, 0)$

$$m \text{ (gradient)} = \frac{0 - \frac{9}{4}}{\frac{9}{2} - 3} = \frac{-3}{2}$$

$$\text{Equation: } y - y_1 = m(x - x_1)$$

$$y - \frac{9}{4} = \frac{-3}{2}(x - 3)$$

$$L \therefore y = \frac{-3}{2}x + \frac{27}{4}$$

$$\text{Equation of a Line: } y - y_1 = m(x - x_1)$$

② Work out the inequalities now

$$1 - y < \frac{-3}{2}x + \frac{27}{4}$$

$$2 - y < 6x - \frac{27}{4} - x^2$$

$$3 - y > \cos\left(\frac{1}{3}\pi x\right)$$

Sub in our
value of k here

(Total for Question 9 is 13 marks)

TOTAL FOR PAPER IS 75 MARKS

